



Kernels and closures of soft set relations, and soft set relation mappings

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ABSTRACT

In this paper, the notions of anti-reflexive kernel, symmetric kernel, reflexive closure, and symmetric closure of a soft set relation are first introduced, respectively. Then, their accurate calculation formulae and some properties are obtained. Finally, soft set relation mappings and inverse soft set relation mappings are proposed, and some related properties are discussed.

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1. Introduction

To solve complicated problems in economics, engineering and environment, we cannot successfully use classical methods because of different kinds of incomplete knowledge. A wide range of theories such as probability theory, fuzzy set theory, intuitionistic fuzzy set theory, rough set theory, vague set theory and the interval mathematics are well known and often useful mathematical approaches for modeling uncertainty. However, what these theories can handle is merely a proper part of uncertainty. Each of these theories has its inherent difficulties as pointed out by Molodtsov [1]. The reason for these difficulties is, possibly, the inadequacy of the parametrization tool of these theories. Molodtsov [1] initiated the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the existing theoretical approaches. This theory has proven useful in many different fields such as decision making [2–7], data analysis [8], forecasting [9], simulation [10], and evaluation of sound quality [11].

The concept and basic properties of soft set theory were presented in [1,12]. In the classical soft set theory, a situation may be complex in the real world because of the fuzzy nature of the parameters. With this point of view, the classical soft sets have been extended to fuzzy soft sets [13,14], intuitionistic fuzzy soft sets [15–17], vague soft sets [18], interval-valued fuzzy soft sets [19], and interval-valued intuitionistic fuzzy soft sets [20], respectively. Up to the present, soft set theory has been applied to several algebra structures: groups [21,22], semirings [23], rings [24], BCK/BCI-algebras [25–27], d -algebras [28], ordered semigroups [29], and BL-algebras [30]. Xiao et al. [31] proposed the notion of exclusive disjunctive soft sets and studied some of its operations. Gong et al. [32] studied the bijective soft set with its operations. Ontology-based (or DL-based) soft set theory was presented in [33].

Algebraic structures of soft sets have been studied by some authors. Maji et al. [12] presented some definitions on soft sets such as a soft subset, the complement of a soft set. Based on the analysis of several operations on soft sets introduced in [12], Ali et al. [34] presented some new algebraic operations for soft sets and proved that certain De Morgan's laws hold

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in soft set theory with respect to these new definitions. Qin and Hong introduced the concept of soft equality and some related properties are derived in [35]. Kharal and Ahmad [36] introduced the notion of a mapping on the classes of fuzzy soft sets which is a pivotal notion for the advanced development of any new area of mathematical sciences. In [4], Çağman and Enginoğlu studied products of soft sets and *uni-int* decision function. Recently, Babitha and Sunil [37] proposed soft set relations and many related concepts are discussed. In the present paper, we attempt to conduct a further study along this line.

This paper is an attempt to broad the theoretical aspects of soft sets again. In order to refresh the fundamental concepts of set theory we refer to [38,39]. The rest of this paper is organized as follows. The following section briefly recalls the notions of soft sets and soft set relations. In Section 3, we define anti-reflexive kernel and symmetric kernel of a soft set relation, respectively. Results involving them are obtained. Section 4 gives the concepts of reflexive closure and symmetric closure of a soft set relation, and proves some theorems based on them. In Section 5, we propose soft set relation mappings and inverse soft set relation mappings, and some related properties are discussed. The last section summarizes the conclusions and presents some topics for future research.

2. Preliminaries

In this section we will briefly recall the notions of soft sets, and soft set relations. See especially [1,12,37] for further details and background.

Definition 2.1 ([1]). Let U be an initial universe set and E be a set of parameters. Let $\mathcal{P}(U)$ denote the power set of U and $A \subset E$. A pair (F, A) is called a soft set over U , where F is a mapping given by $F : A \longrightarrow \mathcal{P}(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U . For each $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε -approximate elements of the soft set (F, A) .

Definition 2.2 ([12]). For two soft sets (F, A) and (G, B) over a common universe U , we say that (F, A) is a soft subset of (G, B) if

- (i) $A \subset B$, and
- (ii) $\forall \varepsilon \in A$, $F(\varepsilon)$ and $G(\varepsilon)$ are identical approximations.

We write $(F, A) \widetilde{\subset} (G, B)$.

(F, A) is said to be a soft superset of (G, B) , if (G, B) is a soft subset of (F, A) . We denote it by $(F, A) \widetilde{\supset} (G, B)$. Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A) .

Definition 2.3 ([12]). Let $E = \{e_1, e_2, e_3, \dots, e_n\}$ be a set of parameters. The NOT set of E denoted by $\neg E$ is defined by, $\neg E = \{\neg e_1, \neg e_2, \neg e_3, \dots, \neg e_n\}$, where $\forall i$, $\neg e_i = \text{not } e_i$.

Definition 2.4 ([12]). The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \neg A)$, where $F^c : \neg A \longrightarrow \mathcal{P}(U)$ is a mapping given by $F^c(\neg \alpha) = U - F(\alpha)$, $\forall \neg \alpha \in \neg A$.

Definition 2.5 ([12]). A soft set (F, A) over U is said to be a NULL soft set denoted by Φ , if $\forall \varepsilon \in A$, $F(\varepsilon) = \emptyset$, (null-set).

A soft set (F, A) over U is said to be an absolute soft set denoted by \bar{A} , if $\forall \varepsilon \in A$, $F(\varepsilon) = U$.

Proposition 2.6 ([12]). If A and B are two sets of parameters then we have the following

- (1) $\neg(\neg A) = A$;
- (2) $\neg(A \cup B) = \neg A \cap \neg B$;
- (3) $\neg(A \cap B) = \neg A \cup \neg B$.

Definition 2.7 ([12]). A union of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C) , where $C = A \cup B$, and $\forall e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A) \widetilde{\cup} (G, B) = (H, C)$.

Definition 2.8 ([40]). An intersection of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C) , where $C = A \cap B$, and $\forall e \in C$, $H(e) = F(e) \cap G(e)$.

Definition 2.9 ([37]). Let (F, A) and (G, B) be two soft sets over U , then the Cartesian product of (F, A) and (G, B) is defined as, $(F, A) \times (G, B) = (H, A \times B)$, where $H : A \times B \longrightarrow \mathcal{P}(U \times U)$ and $H(a, b) = F(a) \times G(b)$, where $(a, b) \in A \times B$. i.e., $H(a, b) = \{(h_i, h_j) : \text{where } h_i \in F(a) \text{ and } h_j \in G(b)\}$.

The Cartesian product of three or more nonempty soft sets can be defined by generalizing the definition of the Cartesian product of two soft sets. The Cartesian product $(F_1, A) \times (F_2, A) \times \cdots \times (F_n, A)$ of the nonempty soft sets $(F_1, A), (F_2, A), \dots, (F_n, A)$ is the soft set of all ordered n -tuple $\{h_1, h_2, \dots, h_n\}$ where $h_i \in F_i(a)$.

Example 2.10 ([37]). Consider the soft set (F, A) which describes the “cost of the houses” and the soft set (G, B) which describes the “attractiveness of the houses”.

Suppose that $U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}\}$,

$A = \{\text{very costly; costly; cheap}\}$ and

$B = \{\text{beautiful; in the green surroundings; cheap}\}$.

Let $F(\text{very costly}) = \{h_2, h_4, h_7, h_8\}$,

$F(\text{costly}) = \{h_1, h_3, h_5\}$,

$F(\text{cheap}) = \{h_6, h_9, h_{10}\}$, and

$G(\text{beautiful}) = \{h_2, h_3, h_7\}$,

$G(\text{in the green surroundings}) = \{h_6, h_5, h_8\}$,

$G(\text{cheap}) = \{h_6, h_9, h_{10}\}$.

Now $(F, A) \times (G, B) = (H, A \times B)$ where a typical element will look like

$$\begin{aligned} H(\text{very costly, beautiful}) &= \{h_2, h_4, h_7, h_8\} \times \{h_2, h_3, h_7\} \\ &= \{(h_2, h_2), (h_2, h_3), (h_2, h_7), (h_4, h_2), (h_4, h_3), (h_4, h_7), \\ &\quad (h_7, h_2), (h_7, h_3), (h_7, h_7), (h_8, h_2), (h_8, h_3), (h_8, h_7)\}. \end{aligned}$$

Definition 2.11 ([37]). Let (F, A) and (G, B) be two soft sets over U , then a soft set relation from (F, A) to (G, B) is a soft subset of $(F, A) \times (G, B)$.

In other words, a soft set relation from (F, A) to (G, B) is the form (H_1, S) where $S \subset A \times B$ and $H_1(a, b) = H(a, b)$, $\forall (a, b) \in S$ where $(H, A \times B) = (F, A) \times (G, B)$ as defined in Definition 2.9. Any subset of $(F, A) \times (F, A)$ is called a soft set relation on (F, A) .

In an equivalent way, we can define the soft set relation R on the soft set (F, A) in the parameterized form as follows:

If $(F, A) = \{F(a), F(b), \dots\}$, then $F(a)RF(b)$ iff $F(a) \times F(b) \in R$.

A soft set relation on a soft set can be regarded as a special set. $\mathcal{SSR}(F, A)$ will denote the family of all soft set relations on (F, A) .

3. Kernels of soft set relations

In this section, we will introduce the notions of anti-reflexive kernel and symmetric kernel of a soft set relation, and study their properties.

Definition 3.1 ([37]). Let R be a soft set relation on (F, A) , then

- (1) R is reflexive if $H_1(a, a) \in R, \forall a \in A$.
- (2) R is symmetric if $H_1(a, b) \in R \implies H_1(b, a) \in R, \forall (a, b) \in A \times A$.

Next, we will give their equivalent definitions, respectively.

Definition 3.2. Let R be a soft set relation on (F, A) , then

- (1) R is reflexive if $F(a) \times F(a) \in R, \forall a \in A$.
- (2) R is symmetric if $F(a) \times F(b) \in R \implies F(b) \times F(a) \in R, \forall (a, b) \in A \times A$.

Definition 3.3. Let R be a soft set relation on (F, A) , then R is anti-reflexive if $F(a) \times F(a) \notin R, \forall a \in A$.

Definition 3.4. Let I be a soft set relation on (F, A) . If $\forall a, b \in A$ and $a \neq b, F(a) \times F(a) \in I$, but $F(a) \times F(b) \notin I$, then I is called the identity soft set relation.

In what follows, we will denote the identity soft set relation by I later on.

- Definition 3.5.** (1) The inverse of a soft set relation R on (F, A) denoted as R^{-1} is defined by $R^{-1} = \{F(b) \times F(a) \mid F(a) \times F(b) \in R\}$ (see [37]).
- (2) The complement of a soft set relation R on (F, A) denoted as R^c is defined by $R^c = \{F(a) \times F(b) \mid F(a) \times F(b) \notin R, a, b \in A\}$.
- (3) The union of two soft set relations R and Q on (F, A) denoted as $R \cup Q$ is defined by $R \cup Q = \{F(a) \times F(b) \mid F(a) \times F(b) \in R \text{ or } F(a) \times F(b) \in Q\}$.
- (4) The intersection of two soft set relations R and Q on (F, A) denoted as $R \cap Q$ is defined by $R \cap Q = \{F(a) \times F(b) \mid F(a) \times F(b) \in R \text{ and } F(a) \times F(b) \in Q\}$.

Example 3.6. Consider a soft set (F, A) over U where $U = \{h_1, h_2, h_3, h_4\}$, $A = \{m_1, m_2\}$ and $F(m_1) = \{h_1, h_2\}$, $F(m_2) = \{h_2, h_3, h_4\}$. A soft set relation R on (F, A) is defined as

$$R = \{F(m_1) \times F(m_1), F(m_2) \times F(m_1)\}.$$

$$R^c = \{F(m_1) \times F(m_2), F(m_2) \times F(m_2)\}.$$

Another soft set relation Q on (F, A) is defined as

$$Q = \{F(m_1) \times F(m_1), F(m_2) \times F(m_2)\}.$$

$$R \cup Q = \{F(m_1) \times F(m_1), F(m_2) \times F(m_1), F(m_2) \times F(m_2)\},$$

$$R \cap Q = \{F(m_1) \times F(m_1)\}.$$

It is easy to verify that the union and the intersection of soft set relations satisfy commutative law, associative law, and distributive law.

Definition 3.7. Let R, Q be two soft set relations on (F, A) . $\forall a, b \in A$, if $F(a) \times F(b) \in R \implies F(a) \times F(b) \in Q$, then we call $R \subset Q$ (or $R \leq Q$).

Theorem 3.8. Let R, Q be two soft set relations on (F, A) . Then

- (1) R is symmetric iff $R = R^{-1}$.
- (2) $(R^c)^{-1} = (R^{-1})^c$.
- (3) $(R^{-1})^{-1} = R$, $(R^c)^c = R$.
- (4) $R \cup Q \supset R$, $R \cup Q \supset Q$.
- (5) $R \cap Q \subset R$, $R \cap Q \subset Q$.
- (6) $R \subset Q \implies R^{-1} \subset Q^{-1}$.
- (7) If $P \supset Q$ and $P \supset R$, then $P \supset R \cup Q$.
- (8) If $P \subset Q$ and $P \subset R$, then $P \subset R \cap Q$.
- (9) If $R \subset Q$, then $R \cup Q = Q$ and $R \cap Q = R$.
- (10) $(R \cup Q)^{-1} = R^{-1} \cup Q^{-1}$, $(R \cap Q)^{-1} = R^{-1} \cap Q^{-1}$.
- (11) $(R \cup Q)^c = R^c \cap Q^c$, $(R \cap Q)^c = R^c \cup Q^c$.

Proof. Obviously, (3)–(9) hold. We only show (1), (2), (10), and (11).

(1) “ \implies ” $\forall F(a) \times F(b) \in R$, by the symmetry of R , we have $F(b) \times F(a) \in R$, then $F(a) \times F(b) \in R^{-1}$. So $R \subset R^{-1}$. Conversely, $\forall F(a) \times F(b) \in R^{-1}$, then $F(b) \times F(a) \in R$. By the symmetry of R , $F(a) \times F(b) \in R$. So $R^{-1} \subset R$. Thus $R = R^{-1}$.

“ \impliedby ” Assume $R = R^{-1}$. $\forall a, b \in A$, if $F(a) \times F(b) \in R$, then $F(a) \times F(b) \in R^{-1}$, so $F(b) \times F(a) \in R$. Hence R is symmetric. (2) $\forall F(a) \times F(b) \in (R^c)^{-1}$, $a, b \in A$, then $F(b) \times F(a) \in R^c$, which implies that $F(b) \times F(a) \notin R$. Thus $F(a) \times F(b) \notin R^{-1}$, which implies that $F(a) \times F(b) \in (R^{-1})^c$. Hence $(R^c)^{-1} \subset (R^{-1})^c$. Conversely, $\forall F(a) \times F(b) \in (R^{-1})^c$, $a, b \in A$, then $F(a) \times F(b) \notin R^{-1}$, which implies that $F(b) \times F(a) \notin R$. So $F(b) \times F(a) \in R^c$, which implies that $F(a) \times F(b) \in (R^c)^{-1}$. Hence $(R^{-1})^c \subset (R^c)^{-1}$. Therefore $(R^c)^{-1} = (R^{-1})^c$.

(10) $\forall a, b \in A$, $F(a) \times F(b) \in (R \cup Q)^{-1} \iff F(b) \times F(a) \in R \cup Q \iff F(b) \times F(a) \in R \text{ or } F(b) \times F(a) \in Q \iff F(a) \times F(b) \in R^{-1} \text{ or } F(a) \times F(b) \in Q^{-1} \iff F(a) \times F(b) \in R^{-1} \cup Q^{-1}$. So $(R \cup Q)^{-1} = R^{-1} \cup Q^{-1}$. The proof of $(R \cap Q)^{-1} = R^{-1} \cap Q^{-1}$ is similar.

(11) $\forall a, b \in A$, $F(a) \times F(b) \in (R \cup Q)^c \iff F(a) \times F(b) \notin R \cup Q \iff F(a) \times F(b) \notin R \text{ and } F(a) \times F(b) \notin Q \iff F(a) \times F(b) \in R^c \text{ and } F(a) \times F(b) \in Q^c \iff F(a) \times F(b) \in R^c \cap Q^c$. So $(R \cup Q)^c = R^c \cap Q^c$. The proof of $(R \cap Q)^c = R^c \cup Q^c$ is similar. \square

According to Theorem 3.8(1) and (2), the complement of a symmetric soft set relation is also a symmetric soft set relation.

For convenience, m (resp., M) will denote the minimal (resp., the maximal) soft set relation on (F, A) , then $m = \emptyset$, and $M = (F, A) \times (F, A) = \{F(a) \times F(b) \mid a, b \in A\}$. It is easy to verify that m and I^c are two anti-reflexive soft set relations, m, M , and I are three symmetric soft set relations, M and I are two reflexive soft set relations. If R is not an anti-reflexive soft set relation, then there is no anti-reflexive soft set relation containing R . If R is not a reflexive soft set relation, then there is no reflexive soft set relation contained in R . Moreover, if R is a reflexive soft set relation, then $R \supset I$, and if R is an anti-reflexive soft set relation, then $R \subset I^c$.

Definition 3.9. Let R be a soft set relation on (F, A) .

- (1) The maximal anti-reflexive soft set relation contained in R is called anti-reflexive kernel of R , denoted by $ar(R)$.
- (2) The maximal symmetric soft set relation contained in R is called symmetric kernel of R , denoted by $s(R)$.

Theorem 3.10. Let R be a soft set relation on (F, A) . Then

- (1) $ar(R) = R \cap I^c$. Therefore we obtain a mapping (called anti-reflexive kernel operator) $ar : \mathcal{SSR}(F, A) \longrightarrow \mathcal{SSR}(F, A)$.
- (2) $s(R) = R \cap R^{-1}$. Therefore we obtain a mapping (called symmetric kernel operator) $s : \mathcal{SSR}(F, A) \longrightarrow \mathcal{SSR}(F, A)$.

Proof. (1) By Theorem 3.8(5), $R \cap I^c \subset R$ and $R \cap I^c \subset I^c$.

Step 1: $\forall a \in A$, by the definition of I , $F(a) \times F(a) \in I$, so $F(a) \times F(a) \notin I^c$. Hence $F(a) \times F(a) \notin R \cap I^c$, i.e. $R \cap I^c$ is an anti-reflexive soft set relation on (F, A) by Definition 3.3.

Step 2: If T is an anti-reflexive soft set relation on (F, A) and $T \subset R$. Then $T \subset I^c$. Hence $T \subset R \cap I^c$.

So $ar(R) = R \cap I^c$.

(2) By Theorem 3.8(10) and (3), $(R \cap R^{-1})^{-1} = R^{-1} \cap (R^{-1})^{-1} = R^{-1} \cap R = R \cap R^{-1}$.

i.e. $R \cap R^{-1}$ is a symmetric soft set relation on (F, A) by Theorem 3.8(1). According to Theorem 3.8(5), $R \cap R^{-1} \subset R$. On the other hand, if T is a symmetric soft set relation on (F, A) and $T \subset R$. By Theorem 3.8(6), $T^{-1} \subset R^{-1}$. Then by Theorem 3.8(1) and (8), $T = T^{-1} \subset R \cap R^{-1}$. So $s(R) = R \cap R^{-1}$. \square

Example 3.11. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$, $A = \{e_1, e_2, e_3\}$. The soft set (F, A) is given by $\{F(e_1) = \{u_1, u_2\}, F(e_2) = \{u_2, u_3, u_4\}, F(e_3) = \{u_1, u_5, u_6, u_7\}\}$. A soft set relation R on (F, A) is given by $R = \{F(e_1) \times F(e_1), F(e_2) \times F(e_3), F(e_3) \times F(e_1)\}$. Then

$$\begin{aligned} ar(R) &= R \cap I^c \\ &= \{F(e_1) \times F(e_1), F(e_2) \times F(e_3), F(e_3) \times F(e_1)\} \\ &\quad \cap \{F(e_1) \times F(e_2), F(e_1) \times F(e_3), F(e_2) \times F(e_1), F(e_2) \times F(e_3), F(e_3) \times F(e_1), F(e_3) \times F(e_2)\} \\ &= \{F(e_2) \times F(e_3), F(e_3) \times F(e_1)\} \\ s(R) &= R \cap R^{-1} = \{F(e_1) \times F(e_1), F(e_2) \times F(e_3), F(e_3) \times F(e_1)\} \\ &\quad \cap \{F(e_1) \times F(e_1), F(e_3) \times F(e_2), F(e_1) \times F(e_3)\} \\ &= \{F(e_1) \times F(e_1)\}. \end{aligned}$$

Theorem 3.12. The anti-reflexive kernel operator ar has the following properties:

- (1) $ar(m) = m$, $ar(I^c) = I^c$.
- (2) $\forall R \in \mathcal{SSR}(F, A)$, $ar(R) \subset R$.
- (3) $\forall R, Q \in \mathcal{SSR}(F, A)$, $ar(R \cup Q) = ar(R) \cup ar(Q)$, $ar(R \cap Q) = ar(R) \cap ar(Q)$.
- (4) $\forall R, Q \in \mathcal{SSR}(F, A)$, if $R \subset Q$, then $ar(R) \subset ar(Q)$.
- (5) $\forall R \in \mathcal{SSR}(F, A)$, $ar(ar(R)) = ar(R)$.

Proof. (1) By the anti-reflexivity of m and I^c , obviously, $ar(m) = m$, $ar(I^c) = I^c$.

(2) $\forall R \in \mathcal{SSR}(F, A)$, by Theorems 3.10(1) and 3.8(5), $ar(R) = R \cap I^c \subset R$.

(3) $\forall R, Q \in \mathcal{SSR}(F, A)$, by Theorem 3.10(1),

$$\begin{aligned} ar(R \cup Q) &= (R \cup Q) \cap I^c = (R \cap I^c) \cup (Q \cap I^c) = ar(R) \cup ar(Q), \\ ar(R \cap Q) &= (R \cap Q) \cap I^c = (R \cap I^c) \cap (Q \cap I^c) = ar(R) \cap ar(Q). \end{aligned}$$

(4) $\forall R, Q \in \mathcal{SSR}(F, A)$, $R \subset Q$, by (3) and Theorem 3.8(4) and (9), $ar(Q) = ar(R \cup Q) = ar(R) \cup ar(Q) \supset ar(R)$.

(5) $\forall R \in \mathcal{SSR}(F, A)$, by Theorem 3.10(1), $ar(R) = R \cap I^c$. Hence

$$ar(ar(R)) = ar(R \cap I^c) = (R \cap I^c) \cap I^c = R \cap I^c = ar(R). \quad \square$$

Theorem 3.13. The symmetric kernel operator s has the following properties:

- (1) $s(m) = m$, $s(M) = M$, $s(I) = I$.
- (2) $\forall R \in \mathcal{SSR}(F, A)$, $s(R) \subset R$.
- (3) $\forall R, Q \in \mathcal{SSR}(F, A)$, $s(R \cap Q) = s(R) \cap s(Q)$.
- (4) $\forall R, Q \in \mathcal{SSR}(F, A)$, if $R \subset Q$, then $s(R) \subset s(Q)$.
- (5) $\forall R \in \mathcal{SSR}(F, A)$, $s(s(R)) = s(R)$.

Proof. (1) By the symmetry of m , M and I , $s(m) = m$, $s(M) = M$, $s(I) = I$.

(2) $\forall R \in \mathcal{SSR}(F, A)$, by Theorems 3.10(2) and 3.8(5), $s(R) = R \cap R^{-1} \subset R$.

(3) $\forall R, Q \in \mathcal{SSR}(F, A)$, by Theorems 3.10(2) and 3.8(10), we have

$$s(R \cap Q) = (R \cap Q) \cap (R \cap Q)^{-1} = (R \cap Q) \cap (R^{-1} \cap Q^{-1}) = (R \cap R^{-1}) \cap (Q \cap Q^{-1}) = s(R) \cap s(Q).$$

- (4) $\forall R, Q \in \mathcal{SSR}(F, A)$, $R \subset Q$, by (3) and Theorem 3.8(5) and (9), $s(Q) = s(R \cap Q) = s(R) \cap s(Q) \subset s(Q)$.
 (5) $\forall R \in \mathcal{SSR}(F, A)$, by Theorem 3.10(2), $s(R) = R \cap R^{-1}$. Hence

$$\begin{aligned} s(s(R)) &= s(R \cap R^{-1}) = (R \cap R^{-1}) \cap (R \cap R^{-1})^{-1} \\ &= (R \cap R^{-1}) \cap (R^{-1} \cap (R^{-1})^{-1}) \\ &= (R \cap R^{-1}) \cap (R^{-1} \cap R) \\ &= R \cap R^{-1} \\ &= s(R). \quad \square \end{aligned}$$

According to Theorem 3.13, the symmetric kernel operator s is an interior operator in topology [41].

4. Closures of soft set relations

In this section, we will introduce the concepts of reflexive closure and symmetric closure of a soft set relation, and study their properties.

Definition 4.1. Let R be a soft set relation on (F, A) . The minimal reflexive soft set relation containing R is called reflexive closure of R , denoted by $\bar{r}(R)$.

Definition 4.2. Let R be a soft set relation on (F, A) . The minimal symmetric soft set relation containing R is called symmetric closure of R , denoted by $\bar{s}(R)$.

Theorem 4.3. Let R be a soft set relation on (F, A) . Then

- (1) $\bar{r}(R) = R \cup I$. Therefore we obtain a mapping (called reflexive closure operator) $\bar{r} : \mathcal{SSR}(F, A) \longrightarrow \mathcal{SSR}(F, A)$.
 (2) $\bar{s}(R) = R \cup R^{-1}$. Therefore we obtain a mapping (called symmetric closure operator) $\bar{s} : \mathcal{SSR}(F, A) \longrightarrow \mathcal{SSR}(F, A)$.

Proof. (1) By Theorem 3.8(4), $R \cup I \supset R, \forall a \in A, F(a) \times F(a) \in I \subset R \cup I$, so $R \cup I$ is reflexive. On the other hand, if T is a reflexive soft set relation on (F, A) and $T \supset R$. By the reflexivity of T , $T \supset I$, thus by Theorem 3.8(7), we have $T \supset R \cup I$. So $\bar{r}(R) = R \cup I$.

(2) By Theorem 3.8(10), $(R \cup R^{-1})^{-1} = R^{-1} \cup (R^{-1})^{-1} = R^{-1} \cup R = R \cup R^{-1}$, i.e. $R \cup R^{-1}$ is a symmetric soft set relation on (F, A) , and $R \cup R^{-1} \supset R$ by Theorem 3.8(4).

If T is a symmetric soft set relation on (F, A) and $T \supset R$. By Theorem 3.8(6), $T^{-1} \supset R^{-1}$. According to Theorem 3.8(1) and (7), $T = T^{-1} \supset R \cup R^{-1}$. So $\bar{s}(R) = R \cup R^{-1}$. \square

Example 4.4. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$, $A = \{d, e, f, g\}$. The soft set (F, A) is given by $\{F(d) = \{u_1, u_2\}, F(e) = \{u_2, u_3, u_4\}, F(f) = \{u_1, u_5, u_6\}, F(g) = \{u_7\}\}$. Consider a soft set relation R defined on (F, A) as

$$\begin{aligned} R &= \{F(d) \times F(e), F(e) \times F(g), F(f) \times F(f)\}. \text{ Then} \\ \bar{r}(R) &= R \cup I \\ &= \{F(d) \times F(e), F(e) \times F(g), F(f) \times F(f)\} \cup \{F(d) \times F(d), F(e) \times F(e), F(f) \times F(f), F(g) \times F(g)\} \\ &= \{F(d) \times F(d), F(d) \times F(e), F(e) \times F(e), F(e) \times F(g), F(f) \times F(f), F(g) \times F(g)\} \\ \bar{s}(R) &= R \cup R^{-1} = \{F(d) \times F(e), F(e) \times F(g), F(f) \times F(f)\} \cup \{F(e) \times F(d), F(g) \times F(e), F(f) \times F(f)\} \\ &= \{F(d) \times F(e), F(e) \times F(g), F(e) \times F(d), F(g) \times F(e), F(f) \times F(f)\}. \end{aligned}$$

Theorem 4.5. The reflexive closure operator \bar{r} has the following properties:

- (1) $\bar{r}(M) = M, \bar{r}(I) = I$.
 (2) $\forall R \in \mathcal{SSR}(F, A)$, $R \subset \bar{r}(R)$.
 (3) $\forall R, Q \in \mathcal{SSR}(F, A)$, $\bar{r}(R \cup Q) = \bar{r}(R) \cup \bar{r}(Q)$, $\bar{r}(R \cap Q) = \bar{r}(R) \cap \bar{r}(Q)$.
 (4) $\forall R, Q \in \mathcal{SSR}(F, A)$, if $R \subset Q$, then $\bar{r}(R) \subset \bar{r}(Q)$.
 (5) $\forall R \in \mathcal{SSR}(F, A)$, $\bar{r}(\bar{r}(R)) = \bar{r}(R)$.

Proof. (1) By the reflexivity of M and I , $\bar{r}(M) = M, \bar{r}(I) = I$.

(2) $\forall R \in \mathcal{SSR}(F, A)$, by Theorems 4.3(1) and 3.8(4), $\bar{r}(R) = R \cup I \supset R$.

(3) $\forall R, Q \in \mathcal{SSR}(F, A)$, by Theorem 4.3(1),

$$\begin{aligned} \bar{r}(R \cup Q) &= (R \cup Q) \cup I = (R \cup I) \cup (Q \cup I) = \bar{r}(R) \cup \bar{r}(Q), \\ \bar{r}(R \cap Q) &= (R \cap Q) \cup I = (R \cup I) \cap (Q \cup I) = \bar{r}(R) \cap \bar{r}(Q). \end{aligned}$$

- (4) $\forall R, Q \in \mathcal{SSR}(F, A)$, $R \subset Q$, by (3) and Theorem 3.8(4,9), $\bar{r}(Q) = \bar{r}(R \cup Q) = \bar{r}(R) \cup \bar{r}(Q) \supset \bar{r}(R)$.
 (5) $\forall R \in \mathcal{SSR}(F, A)$, by Theorem 4.3(1), $\bar{r}(R) = R \cup I$. Hence

$$\bar{r}(\bar{r}(R)) = \bar{r}(R \cup I) = (R \cup I) \cup I = R \cup I = \bar{r}(R). \quad \square$$

Theorem 4.6. The symmetric closure operator \bar{s} has the following properties:

- (1) $\bar{s}(m) = m$, $\bar{s}(M) = M$, $\bar{s}(I) = I$.
 (2) $\forall R \in \mathcal{SSR}(F, A)$, $\bar{s}(R) \supset R$.
 (3) $\forall R, Q \in \mathcal{SSR}(F, A)$, $\bar{s}(R \cup Q) = \bar{s}(R) \cup \bar{s}(Q)$.
 (4) $\forall R, Q \in \mathcal{SSR}(F, A)$, if $R \subset Q$, then $\bar{s}(R) \subset \bar{s}(Q)$.
 (5) $\forall R \in \mathcal{SSR}(F, A)$, $\bar{s}(\bar{s}(R)) = \bar{s}(R)$.

Proof. (1) By the symmetry of m , M and I , $\bar{s}(m) = m$, $\bar{s}(M) = M$, $\bar{s}(I) = I$.

(2) $\forall R \in \mathcal{SSR}(F, A)$, by Theorem 4.3(2), $\bar{s}(R) = R \cup R^{-1} \supset R$.

(3) $\forall R, Q \in \mathcal{SSR}(F, A)$, by Theorems 4.3(2) and 3.8(10), we have

$$\begin{aligned} \bar{s}(R \cup Q) &= (R \cup Q) \cup (R \cup Q)^{-1}, \\ &= (R \cup Q) \cup (R^{-1} \cup Q^{-1}) \\ &= (R \cup R^{-1}) \cup (Q \cup Q^{-1}) \\ &= \bar{s}(R) \cup \bar{s}(Q). \end{aligned}$$

(4) $\forall R, Q \in \mathcal{SSR}(F, A)$, $R \subset Q$, by (3) and Theorem 3.8(4,9), $\bar{s}(Q) = \bar{s}(R \cup Q) = \bar{s}(R) \cup \bar{s}(Q) \supset \bar{s}(R)$.

(5) $\forall R \in \mathcal{SSR}(F, A)$, by Theorem 4.3(2), $\bar{s}(R) = R \cup R^{-1}$. Hence

$$\begin{aligned} \bar{s}(\bar{s}(R)) &= \bar{s}(R \cup R^{-1}) = (R \cup R^{-1}) \cup (R \cup R^{-1})^{-1} \\ &= (R \cup R^{-1}) \cup (R^{-1} \cup (R^{-1})^{-1}) \\ &= (R \cup R^{-1}) \cup (R^{-1} \cup R) \\ &= R \cup R^{-1} \\ &= \bar{s}(R). \quad \square \end{aligned}$$

According to Theorem 4.6, the symmetric closure operator \bar{s} is a closure operator in topology [41].

Lemma 4.7. $\forall R \in \mathcal{SSR}(F, A)$, we have

- (1) $(\bar{r}(R^c))^c = ar(R)$.
 (2) $\bar{r}(ar(R)) = \bar{r}(R)$.
 (3) $ar(\bar{r}(R)) = ar(R)$.

Proof. (1) By Theorem 4.3(1), $\bar{r}(R^c) = R^c \cup I$. By Theorems 3.8(11) and 3.10(1), $(\bar{r}(R^c))^c = (R^c \cup I)^c = (R^c)^c \cap I^c = R \cap I^c = ar(R)$.

(2) By Theorems 3.10(1) and 4.3(1), $\bar{r}(ar(R)) = \bar{r}(R \cap I^c) = (R \cap I^c) \cup I = (R \cup I) \cap (I^c \cup I) = (R \cup I) \cap M = \bar{r}(R)$.

(3) By Theorems 3.10(1) and 4.3(1), $ar(\bar{r}(R)) = ar(R \cup I) = (R \cup I) \cap I^c = (R \cap I^c) \cup (I \cap I^c) = (R \cap I^c) \cup m = ar(R)$. \square

Theorem 4.8 (Six-relations Theorem 1). $\forall R \in \mathcal{SSR}(F, A)$, six different soft set relations are the most soft set relations that can be obtained by using anti-reflexive kernel operator, reflexive closure operator, and complement operator.

Proof. $\forall R \in \mathcal{SSR}(F, A)$, by Lemma 4.7(1), $(\bar{r}(R^c))^c = ar(R)$, so we can replace the anti-reflexive kernel operator with the complement operator and the reflexive closure operator.

(1) Take the complement operator first, then the reflexive closure operator on R . The following five different soft set relations are the most soft set relations that can be constructed:

$$R^c, \quad \bar{r}(R^c), \quad (\bar{r}(R^c))^c, \quad \bar{r}((\bar{r}(R^c))^c), \quad (\bar{r}((\bar{r}(R^c))^c))^c.$$

It is because that by Lemma 4.7(1, 2) and Theorem 3.8(3),

$$\bar{r}((\bar{r}((\bar{r}(R^c))^c))^c) = \bar{r}((\bar{r}(ar(R)))^c) = \bar{r}((\bar{r}(R))^c) = \bar{r}(ar(R^c)) = \bar{r}(R^c),$$

which implies that the sixth is the same as the second. This is repeated emergence.

(2) Take the reflexive closure operator first, then the complement operator on R . By Lemma 4.7(1) and (2), $\bar{r}(R) = \bar{r}(ar(R)) = \bar{r}((\bar{r}(R^c))^c)$, which implies that the first is the same as the fourth in (1). This is repeated emergence.

(3) Take the reflexive closure operator successively or the complement operator successively on R . By Theorems 3.8(3) and 4.5(5), $(R^c)^c = R$, $\bar{r}(\bar{r}(R)) = \bar{r}(R)$. This is repeated emergence.

The proof of Theorem 4.8 is complete. \square

The following example is presented to illustrate the basic idea developed in [Theorem 4.8](#).

Example 4.9. Let $U = \{s_1, s_2, s_3, s_4, s_5\}$, $A = \{a, b\}$. The soft set (F, A) is given by $\{F(a) = \{s_1, s_3, s_4\}, F(b) = \{s_2, s_3, s_5\}\}$. A soft set relation R on (F, A) is given by $R = \{F(a) \times F(a), F(b) \times F(a)\}$. By using anti-reflexive kernel operator, reflexive closure operator, and complement operator, the following six different soft set relations are the most soft set relations that can be obtained:

$$\begin{aligned} R^c &= \{F(a) \times F(b), F(b) \times F(b)\}, \\ \bar{R}(R^c) &= R^c \cup I = \{F(a) \times F(a), F(a) \times F(b), F(b) \times F(b)\}, \\ (\bar{R}(R^c))^c &= \{F(b) \times F(a)\}, \\ \bar{R}((\bar{R}(R^c))^c) &= \{F(b) \times F(a), F(a) \times F(a), F(b) \times F(b)\}, \\ (\bar{R}((\bar{R}(R^c))^c))^c &= \{F(a) \times F(b)\}, \\ R &= (R^c)^c = \{F(a) \times F(a), F(b) \times F(a)\}. \end{aligned}$$

Lemma 4.10. $\forall R \in \mathcal{SSR}(F, A)$, we have

- (1) $(\bar{s}(R^c))^c = s(R)$.
- (2) $\bar{s}(s(R)) = s(R)$.
- (3) $s(\bar{s}(R)) = \bar{s}(R)$.

Proof. (1) By [Theorem 4.3\(2\)](#), $\bar{s}(R^c) = R^c \cup (R^c)^{-1}$. By [Theorems 3.8](#) and [3.10\(2\)](#), $(\bar{s}(R^c))^c = (R^c \cup (R^c)^{-1})^c = (R^c)^c \cap ((R^{-1})^c)^c = R \cap R^{-1} = s(R)$.

(2) and (3) The proofs are straightforward and follow from the definitions of symmetric kernel and symmetric closure. \square

Theorem 4.11 (Six-relations Theorem 2). $\forall R \in \mathcal{SSR}(F, A)$, six different soft set relations are the most soft set relations that can be obtained by using symmetric kernel operator, symmetric closure operator, and complement operator.

Proof. $\forall R \in \mathcal{SSR}(F, A)$, by [Lemma 4.7\(1\)](#), $(\bar{s}(R^c))^c = s(R)$, so we can replace the symmetric kernel operator with the complement operator and the symmetric closure operator.

(1) Take the complement operator first, then the symmetric closure operator on R . The following three different soft set relations are the most soft set relations that can be constructed:

$R^c, \bar{s}(R^c), (\bar{s}(R^c))^c$. It is because that by [Theorem 3.8\(3\)](#) and [Lemma 4.10](#),

$\bar{s}((\bar{s}(R^c))^c) = \bar{s}(s(R)) = s(R) = (\bar{s}(R^c))^c$, i.e. the fourth is the same as the third. $(\bar{s}((\bar{s}(R^c))^c))^c = (s(R))^c = \bar{s}(R^c)$, i.e. the fifth is the same as the second. This is repeated emergence.

(2) Take the symmetric closure operator first, then the complement operator on R . The following two different soft set relations are the most soft set relations that can be constructed:

$\bar{s}(R), (\bar{s}(R))^c$. It is because that by [Theorem 3.8\(3\)](#) and [Lemma 4.10](#),

$\bar{s}((\bar{s}(R))^c) = \bar{s}(s(R^c)) = s(R^c) = (\bar{s}(R))^c$, i.e. the third is the same as the second.

$(\bar{s}((\bar{s}(R))^c))^c = ((\bar{s}(R))^c)^c = \bar{s}(R)$, i.e. the fourth is the same as the first. This is repeated emergence.

(3) Take the symmetric closure operator successively or the complement operator successively on R . By [Theorems 3.8\(3\)](#) and [4.5\(5\)](#), $(R^c)^c = R, \bar{s}(\bar{s}(R)) = \bar{s}(R)$. This is repeated emergence.

The proof of [Theorem 4.11](#) is complete. \square

The following example is employed to illustrate the basic idea developed in [Theorem 4.11](#).

Example 4.12. Let (F, A) and R be the soft set and the soft set relation given in [Example 4.9](#), respectively. By using symmetric kernel operator, symmetric closure operator, and complement operator, the following six different soft set relations are the most soft set relations that can be obtained:

$$\begin{aligned} R^c &= \{F(a) \times F(b), F(b) \times F(b)\}, \\ \bar{s}(R^c) &= R^c \cup (R^c)^{-1} = \{F(a) \times F(b), F(b) \times F(a), F(b) \times F(b)\}, \\ (\bar{s}(R^c))^c &= \{F(a) \times F(a)\}, \\ \bar{s}(R) &= R \cup R^{-1} = \{F(a) \times F(a), F(a) \times F(b), F(b) \times F(a)\}, \\ (\bar{s}(R))^c &= \{F(b) \times F(b)\}, \\ R &= (R^c)^c = \{F(a) \times F(a), F(b) \times F(a)\}. \end{aligned}$$

5. Soft set relation mappings

In this section, we will introduce the notions of soft set relation mappings and inverse soft set relation mappings, and discuss some related properties.

Definition 5.1 ([37]). Let R be a soft set relation from (F, A) to (G, B) . Then the domain of R ($\text{dom } R$) is defined as the soft set (D, A_1) where $A_1 = \{a \in A \mid H(a, b) \in R \text{ for some } b \in B\}$ and $D(a_1) = F(a_1) \ \forall a_1 \in A_1$.

The range of R ($\text{ran } R$) is defined as the soft set (RG, B_1) , where $B_1 \subset B$ and $B_1 = \{b \in B \mid H(a, b) \in R \text{ for some } a \in A\}$ and $RG(b_1) = G(b_1) \ \forall b_1 \in B_1$.

Definition 5.2 ([37]). Let (F, A) and (G, B) be two nonempty soft sets. Then a soft set relation “ f ” from (F, A) to (G, B) is called a soft set function if every element in domain has a unique element in the range. If $F(a)fG(b)$ then we write $f(F(a)) = G(b)$.

Definition 5.3 ([37]). A soft set function f from (F, A) to (G, B) is called

- (1) injective (one-one) if $F(a) \neq F(b)$ implies $f(F(a)) \neq f(F(b))$, i.e. each element of the $\text{ran } f$ appears exactly once in the function.
- (2) surjective (onto) if $\text{ran } f = (G, B)$.
- (3) bijective if f is both injective and surjective.

Definition 5.4. Let f be a soft set function from (F, A) to (G, B) ,

- (1) The soft set relation mapping induced by f , denoted by the notation f^\rightarrow , is a mapping from $\mathcal{SSR}(F, A)$ to $\mathcal{SSR}(G, B)$ that maps R to $f^\rightarrow(R)$, where $f^\rightarrow(R)$ is defined by, $f^\rightarrow(R) = \{f(F(a_1)) \times f(F(a_2)) \mid F(a_1) \times F(a_2) \in R\}$.
- (2) The inverse soft set relation mapping induced by f , denoted by the notation f^\leftarrow , is a mapping from $\mathcal{SSR}(G, B)$ to $\mathcal{SSR}(F, A)$ that maps T to $f^\leftarrow(T)$, where $f^\leftarrow(T)$ is defined by, $f^\leftarrow(T) = \{F(a_1) \times F(a_2) \mid f(F(a_1)) \times f(F(a_2)) \in T\}$.

Example 5.5. Let $U = \{p_1, p_2, p_3, p_4, p_5, p_6\}$, $A = \{a_1, a_2, a_3, a_4\}$, and $B = \{b_1, b_2\}$. Consider the soft sets (F, A) and (G, B) defined by $F(a_1) = \{p_1, p_2, p_5\}$, $F(a_2) = \{p_2, p_3, p_4\}$, $F(a_3) = \{p_1, p_2\}$, $F(a_4) = \{p_5, p_6\}$, and $G(b_1) = \{p_1, p_2\}$, $G(b_2) = \{p_2, p_5, p_6\}$. A soft set function f from (F, A) to (G, B) is given by $f = \{F(a_1) \times G(b_1), F(a_2) \times G(b_1), F(a_3) \times G(b_2), F(a_4) \times G(b_2)\}$.

- (1) Let R be a soft set relation on (F, A) given by $R = \{F(a_1) \times F(a_2), F(a_3) \times F(a_2), F(a_4) \times F(a_1)\}$. Then $f^\rightarrow(R) = \{G(b_1) \times G(b_1), G(b_2) \times G(b_1)\}$.
- (2) Let T be a soft set relation on (G, B) given by $T = \{G(b_1) \times G(b_1), G(b_2) \times G(b_2)\}$. Then $f^\leftarrow(T) = \{F(a_1) \times F(a_1), F(a_1) \times F(a_2), F(a_2) \times F(a_2), F(a_2) \times F(a_1), F(a_3) \times F(a_3), F(a_3) \times F(a_4), F(a_4) \times F(a_3), F(a_4) \times F(a_4)\}$.

Next, we will discuss some basic properties of soft set relation mappings and inverse soft set relation mappings.

Theorem 5.6. Let f be a soft set function from (F, A) to (G, B) . $R, Q \in \mathcal{SSR}(F, A)$. Then

- (1) $R \subset Q \implies f^\rightarrow(R) \subset f^\rightarrow(Q)$.
- (2) $f^\rightarrow(R \cup Q) = f^\rightarrow(R) \cup f^\rightarrow(Q)$.
- (3) $f^\rightarrow(R \cap Q) \subset f^\rightarrow(R) \cap f^\rightarrow(Q)$. If f is one-one, then $f^\rightarrow(R \cap Q) = f^\rightarrow(R) \cap f^\rightarrow(Q)$.

Proof. (1) $\forall G(b_1) \times G(b_2) \in f^\rightarrow(R)$, there exists $F(a_1) \times F(a_2) \in R$ such that $G(b_1) \times G(b_2) = f(F(a_1)) \times f(F(a_2))$ by Definition 5.4(1). Since $R \subset Q$, we have $F(a_1) \times F(a_2) \in Q$. Then $G(b_1) \times G(b_2) \in f^\rightarrow(Q)$. So $f^\rightarrow(R) \subset f^\rightarrow(Q)$.

(2) $\forall G(b_1) \times G(b_2) \in f^\rightarrow(R \cup Q)$, there exists $F(a_1) \times F(a_2) \in R \cup Q$ such that $G(b_1) \times G(b_2) = f(F(a_1)) \times f(F(a_2))$ by Definition 5.4(1), i.e. $G(b_1) \times G(b_2) = f(F(a_1)) \times f(F(a_2))$, where $F(a_1) \times F(a_2) \in R$ or $F(a_1) \times F(a_2) \in Q$. Then $G(b_1) \times G(b_2) \in f^\rightarrow(R)$ or $G(b_1) \times G(b_2) \in f^\rightarrow(Q)$. So $f^\rightarrow(R \cup Q) \subset f^\rightarrow(R) \cup f^\rightarrow(Q)$. Conversely, since $R \cup Q \supset R$ and $R \cup Q \supset Q$, we have $f^\rightarrow(R \cup Q) \supset f^\rightarrow(R)$ and $f^\rightarrow(R \cup Q) \supset f^\rightarrow(Q)$ by (1). So $f^\rightarrow(R \cup Q) \supset f^\rightarrow(R) \cup f^\rightarrow(Q)$ by Theorem 3.8(7).

Therefore $f^\rightarrow(R \cup Q) = f^\rightarrow(R) \cup f^\rightarrow(Q)$.

(3) Since $R \cap Q \subset R$ and $R \cap Q \subset Q$, we have $f^\rightarrow(R \cap Q) \subset f^\rightarrow(R)$ and $f^\rightarrow(R \cap Q) \subset f^\rightarrow(Q)$ by (1). So $f^\rightarrow(R \cap Q) \subset f^\rightarrow(R) \cap f^\rightarrow(Q)$ by Theorem 3.8(8).

Conversely, $\forall G(b_1) \times G(b_2) \in f^\rightarrow(R) \cap f^\rightarrow(Q)$, we have $G(b_1) \times G(b_2) \in f^\rightarrow(R)$ and $G(b_1) \times G(b_2) \in f^\rightarrow(Q)$. By Definition 5.4(1), there exists $F(a_1) \times F(a_2) \in R$ such that $G(b_1) \times G(b_2) = f(F(a_1)) \times f(F(a_2))$, and there exists $F((a_1)') \times F((a_2)') \in Q$ such that $G(b_1) \times G(b_2) = f(F(a_1)') \times f(F(a_2)').$ Then $f(F(a_1)) \times f(F(a_2)) = f(F(a_1)') \times f(F(a_2)') = G(b_1) \times G(b_2)$, which implies that $f(F(a_1)) = f(F(a_1)')$ and $f(F(a_2)) = f(F(a_2)').$ If f is one-one, then $F(a_1) = F((a_1)')$ and $F(a_2) = F((a_2)').$ Then $F(a_1) \times F(a_2) = F((a_1)') \times F((a_2)') \in R \cap Q$. By Definition 5.4(1), $G(b_1) \times G(b_2) \in f^\rightarrow(R \cap Q)$. Thus $f^\rightarrow(R) \cap f^\rightarrow(Q) \subset f^\rightarrow(R \cap Q)$. So if f is one-one, then $f^\rightarrow(R \cap Q) = f^\rightarrow(R) \cap f^\rightarrow(Q)$. \square

Theorem 5.7. Let f be a soft set function from (F, A) to (G, B) , $T, S \in \mathcal{SSR}(G, B)$. Then

- (1) $T \subset S \implies f^\leftarrow(T) \subset f^\leftarrow(S)$.
- (2) $f^\leftarrow(T \cup S) = f^\leftarrow(T) \cup f^\leftarrow(S)$.
- (3) $f^\leftarrow(T \cap S) = f^\leftarrow(T) \cap f^\leftarrow(S)$.

Proof. (1) $\forall F(a_1) \times F(a_2) \in f^{\leftarrow}(T), f(F(a_1)) \times f(F(a_2)) \in T$ by Definition 5.4(2). Since $T \subset S$, we have $f(F(a_1)) \times f(F(a_2)) \in S$. Then $F(a_1) \times F(a_2) \in f^{\leftarrow}(S)$. So $f^{\leftarrow}(T) \subset f^{\leftarrow}(S)$.

(2) $\forall F(a_1) \times F(a_2) \in f^{\leftarrow}(T \cup S), f(F(a_1)) \times f(F(a_2)) \in T \cup S$ by Definition 5.4(2), i.e. $f(F(a_1)) \times f(F(a_2)) \in T$ or $f(F(a_1)) \times f(F(a_2)) \in S$. Then $F(a_1) \times F(a_2) \in f^{\leftarrow}(T)$ or $F(a_1) \times F(a_2) \in f^{\leftarrow}(S)$, i.e. $F(a_1) \times F(a_2) \in f^{\leftarrow}(T) \cup f^{\leftarrow}(S)$. So $f^{\leftarrow}(T \cup S) \subset f^{\leftarrow}(T) \cup f^{\leftarrow}(S)$. Conversely, since $T \cup S \supset T$ and $T \cup S \supset S$, we have $f^{\leftarrow}(T \cup S) \supset f^{\leftarrow}(T)$ and $f^{\leftarrow}(T \cup S) \supset f^{\leftarrow}(S)$ by (1). So $f^{\leftarrow}(T \cup S) \supset f^{\leftarrow}(T) \cup f^{\leftarrow}(S)$ by Theorem 3.8(7). Therefore $f^{\leftarrow}(T \cup S) = f^{\leftarrow}(T) \cup f^{\leftarrow}(S)$.

(3) Since $T \cap S \subset T$ and $T \cap S \subset S$, we have $f^{\leftarrow}(T \cap S) \subset f^{\leftarrow}(T)$ and $f^{\leftarrow}(T \cap S) \subset f^{\leftarrow}(S)$ by (1). So $f^{\leftarrow}(T \cap S) \subset f^{\leftarrow}(T) \cap f^{\leftarrow}(S)$ by Theorem 3.8(8).

Conversely, $\forall F(a_1) \times F(a_2) \in f^{\leftarrow}(T) \cap f^{\leftarrow}(S)$, then $F(a_1) \times F(a_2) \in f^{\leftarrow}(T)$ and $F(a_1) \times F(a_2) \in f^{\leftarrow}(S)$. By Definition 5.4(2), $f(F(a_1)) \times f(F(a_2)) \in T$ and $f(F(a_1)) \times f(F(a_2)) \in S$, i.e. $f(F(a_1)) \times f(F(a_2)) \in T \cap S$. Then $F(a_1) \times F(a_2) \in f^{\leftarrow}(T \cap S)$. So $f^{\leftarrow}(T) \cap f^{\leftarrow}(S) \subset f^{\leftarrow}(T \cap S)$. Therefore $f^{\leftarrow}(T \cap S) = f^{\leftarrow}(T) \cap f^{\leftarrow}(S)$. \square

Theorem 5.8. Let f be a soft set function from (F, A) to (G, B) , $R \in \mathcal{SSR}(F, A)$, $T \in \mathcal{SSR}(G, B)$. Then

- (1) $f^{\leftarrow}(f^{\rightarrow}(R)) \supset R$. If f is one-one, then $f^{\leftarrow}(f^{\rightarrow}(R)) = R$.
- (2) $f^{\rightarrow}(f^{\leftarrow}(T)) \subset T$. If f is surjective, then $f^{\rightarrow}(f^{\leftarrow}(T)) = T$.

Proof. (1) $\forall F(a_1) \times F(a_2) \in R$, we have $f(F(a_1)) \times f(F(a_2)) \in f^{\rightarrow}(R)$. By Definition 5.4(2), $F(a_1) \times F(a_2) \in f^{\leftarrow}(f^{\rightarrow}(R))$. So $f^{\leftarrow}(f^{\rightarrow}(R)) \supset R$.

Conversely, $\forall F(a_1) \times F(a_2) \in f^{\leftarrow}(f^{\rightarrow}(R))$, we have $f(F(a_1)) \times f(F(a_2)) \in f^{\rightarrow}(R)$ by Definition 5.4(2). Then there exists $F((a_1)') \times F((a_2)') \in R$ such that $f(F(a_1)) \times f(F(a_2)) = f(F((a_1)')) \times f(F((a_2)'))$, which implies that $f(F(a_1)) = f(F((a_1)'))$ and $f(F(a_2)) = f(F((a_2)'))$. If f is one-one, then $F(a_1) = F((a_1)')$ and $F(a_2) = F((a_2)')$. Thus $F(a_1) \times F(a_2) = F((a_1)') \times F((a_2)') \in R$.

So if f is one-one, then $f^{\leftarrow}(f^{\rightarrow}(R)) = R$.

(2) By Definition 5.4, $\forall G(b_1) \times G(b_2) \in f^{\rightarrow}(f^{\leftarrow}(T))$, then there exists $F(a_1) \times F(a_2) \in f^{\leftarrow}(T)$ such that $G(b_1) \times G(b_2) = f(F(a_1)) \times f(F(a_2))$. Then $G(b_1) \times G(b_2) = f(F(a_1)) \times f(F(a_2)) \in T$. So $f^{\rightarrow}(f^{\leftarrow}(T)) \subset T$.

Conversely, $\forall G(b_1) \times G(b_2) \in T$, if f is surjective, then there exist $F(a_1), F(a_2) \in (F, A)$ such that $G(b_1) = f(F(a_1))$ and $G(b_2) = f(F(a_2))$. Then $f(F(a_1)) \times f(F(a_2)) = G(b_1) \times G(b_2) \in T$, which implies that $F(a_1) \times F(a_2) \in f^{\leftarrow}(T)$. By Definition 5.4(1), $G(b_1) \times G(b_2) = f(F(a_1)) \times f(F(a_2)) \in f^{\rightarrow}(f^{\leftarrow}(T))$.

So if f is surjective, then $f^{\rightarrow}(f^{\leftarrow}(T)) = T$. \square

Theorem 5.9. Let f be a soft set function from (F, A) to (G, B) , $R \in \mathcal{SSR}(F, A)$.

- (1) If f is surjective and R is reflexive, then $f^{\rightarrow}(R)$ is a reflexive soft set relation on (G, B) .
- (2) If f is bijective and R is anti-reflexive, then $f^{\rightarrow}(R)$ is an anti-reflexive soft set relation on (G, B) .
- (3) If R is symmetric, then $f^{\rightarrow}(R)$ is a symmetric soft set relation on (G, B) .

Proof. (1) $\forall G(b) \in (G, B)$, if f is surjective, then there exists $F(a) \in (F, A)$ such that $G(b) = f(F(a))$. If R is reflexive, then $F(a) \times F(a) \in R$, so $G(b) \times G(b) = f(F(a)) \times f(F(a)) \in f^{\rightarrow}(R)$, i.e. $f^{\rightarrow}(R)$ is a reflexive soft set relation on (G, B) .

(2) $\forall G(b) \in (G, B)$, if f is bijective, then there exists $F(a) \in (F, A)$ such that $G(b) = f(F(a))$. If R is anti-reflexive, then $F(a) \times F(a) \notin R$, so $G(b) \times G(b) = f(F(a)) \times f(F(a)) \notin f^{\rightarrow}(R)$, i.e. $f^{\rightarrow}(R)$ is an anti-reflexive soft set relation on (G, B) .

(3) If $G(b_1) \times G(b_2) \in f^{\rightarrow}(R)$, then there exists $F(a_1) \times F(a_2) \in R$ such that $G(b_1) \times G(b_2) = f(F(a_1)) \times f(F(a_2))$, which implies that $G(b_1) = f(F(a_1))$ and $G(b_2) = f(F(a_2))$. If R is symmetric, then $F(a_2) \times F(a_1) \in R$, so $G(b_2) \times G(b_1) = f(F(a_2)) \times f(F(a_1)) \in f^{\rightarrow}(R)$. Therefore $f^{\rightarrow}(R)$ is a symmetric soft set relation on (G, B) . \square

Theorem 5.10. Let f be a soft set function from (F, A) to (G, B) , $T \in \mathcal{SSR}(G, B)$.

- (1) If T is reflexive, then $f^{\leftarrow}(T)$ is a reflexive soft set relation on (F, A) .
- (2) If T is anti-reflexive, then $f^{\leftarrow}(T)$ is an anti-reflexive soft set relation on (F, A) .
- (3) If T is symmetric, then $f^{\leftarrow}(T)$ is a symmetric soft set relation on (F, A) .

Proof. (1) $\forall F(a) \in (F, A)$, then $f(F(a)) \in (G, B)$. If T is reflexive, then $f(F(a)) \times f(F(a)) \in T$, which implies that $F(a) \times F(a) \in f^{\leftarrow}(T)$ by Definition 5.4. So $f^{\leftarrow}(T)$ is a reflexive soft set relation on (F, A) .

(2) $\forall F(a) \in (F, A)$, then $f(F(a)) \in (G, B)$. If T is anti-reflexive, then $f(F(a)) \times f(F(a)) \notin T$, which implies that $F(a) \times F(a) \notin f^{\leftarrow}(T)$ by Definition 5.4. So $f^{\leftarrow}(T)$ is an anti-reflexive soft set relation on (F, A) .

(3) $\forall F(a_1) \times F(a_2) \in f^{\leftarrow}(T)$, then $f(F(a_1)) \times f(F(a_2)) \in T$. If T is symmetric, then $f(F(a_2)) \times f(F(a_1)) \in T$, which implies that $F(a_2) \times F(a_1) \in f^{\leftarrow}(T)$ by Definition 5.4. So $f^{\leftarrow}(T)$ is a symmetric soft set relation on (F, A) . \square

Next, we will give two main results of this section.

Theorem 5.11. Let f be a soft set function from (F, A) to (G, B) , $R \in \mathcal{SSR}(F, A)$. Then

- (1) If f is surjective, then $f^{\rightarrow}(\bar{r}(R)) = \bar{r}(f^{\rightarrow}(R))$.
- (2) $f^{\rightarrow}(\bar{s}(R)) = \bar{s}(f^{\rightarrow}(R))$.
- (3) If f is bijective, then $f^{\rightarrow}(s(R)) = s(f^{\rightarrow}(R))$.
- (4) If f is bijective, then $f^{\rightarrow}(ar(R)) = ar(f^{\rightarrow}(R))$.

Proof. (1) By Theorem 5.9(1) and Definition 4.1, $f^{\rightarrow}(\bar{r}(R))$ is a reflexive soft set relation on (G, B) . By Theorem 4.3(1), $\bar{r}(R) = R \cup I \supset R$. According to Theorem 5.6(1), $f^{\rightarrow}(\bar{r}(R)) \supset f^{\rightarrow}(R)$.

Next, we will show that if W is a reflexive soft set relation on (G, B) and $W \supset f^{\rightarrow}(R)$, then $f^{\rightarrow}(\bar{r}(R)) \subset W$. By Theorems 5.8(1) and 5.7(1), $R \subset f^{\leftarrow}(f^{\rightarrow}(R)) \subset f^{\leftarrow}(W)$ since $f^{\rightarrow}(R) \subset W$, i.e. $R \subset f^{\leftarrow}(W)$. On the other hand, by Theorem 5.10(1), $f^{\leftarrow}(W)$ is a reflexive soft set relation on (F, A) . Then $\bar{r}(R) \subset f^{\leftarrow}(W)$. According to Theorems 5.8(2) and 5.6(1), $f^{\rightarrow}(\bar{r}(R)) \subset f^{\rightarrow}(f^{\leftarrow}(W)) \subset W$, i.e. $f^{\rightarrow}(\bar{r}(R)) \subset W$.

So $f^{\rightarrow}(\bar{r}(R)) = \bar{r}(f^{\rightarrow}(R))$.

(2) It is similar to (1).

(3) By Theorem 5.9(3) and Definition 3.9(2), $f^{\rightarrow}(s(R))$ is a symmetric soft set relation on (G, B) . By Theorem 3.10(2), $s(R) = R \cap R^{-1} \subset R$. According to Theorem 5.6(1), $f^{\rightarrow}(s(R)) \subset f^{\rightarrow}(R)$.

Next, we will show that if W is a symmetric soft set relation on (G, B) and $W \subset f^{\rightarrow}(R)$, then $f^{\rightarrow}(s(R)) \supset W$. Note that f is bijective. By Theorems 5.8(1) and 5.7(1), $R = f^{\leftarrow}(f^{\rightarrow}(R)) \supset f^{\leftarrow}(W)$ since $f^{\rightarrow}(R) \supset W$, i.e. $R \supset f^{\leftarrow}(W)$. On the other hand, by Theorem 5.10(3), $f^{\leftarrow}(W)$ is a symmetric soft set relation on (F, A) . Then $s(R) \supset f^{\leftarrow}(W)$. According to Theorems 5.6(1) and 5.8(2), then $f^{\rightarrow}(s(R)) \supset f^{\rightarrow}(f^{\leftarrow}(W)) = W$, i.e. $f^{\rightarrow}(s(R)) \supset W$.

So $f^{\rightarrow}(s(R)) = s(f^{\rightarrow}(R))$.

(4) It is similar to (3). \square

The following example is presented to illustrate the basic ideas developed in Theorem 5.11.

Example 5.12. Let (F, A) and (G, B) be the soft sets given in Example 5.5, R and f be the soft set relation and the soft set function given in Example 5.5, respectively. Obviously, f is surjective. Then

$$\bar{r}(R) = R \cup I = \{F(a_1) \times F(a_1), F(a_1) \times F(a_2), F(a_2) \times F(a_2), F(a_3) \times F(a_2), \\ F(a_3) \times F(a_3), F(a_4) \times F(a_1), F(a_4) \times F(a_4)\},$$

$$f^{\rightarrow}(\bar{r}(R)) = \{G(b_1) \times G(b_1), G(b_2) \times G(b_1), G(b_2) \times G(b_2)\}.$$

Thus $\bar{r}(f^{\rightarrow}(R)) = \{G(b_1) \times G(b_1), G(b_2) \times G(b_1), G(b_2) \times G(b_2)\}$.

Similarly, we can obtain $\bar{s}(f^{\rightarrow}(R))$ by using $f^{\rightarrow}(\bar{s}(R))$.

Of course, if we define a bijective soft set function f , then we can obtain $s(f^{\rightarrow}(R))$ and $ar(f^{\rightarrow}(R))$ by using $f^{\rightarrow}(s(R))$ and $f^{\rightarrow}(ar(R))$, respectively.

Theorem 5.13. Let f be a soft set function from (F, A) to (G, B) , $T \in \mathcal{SSR}(G, B)$.

- (1) If f is bijective, then $f^{\leftarrow}(\bar{r}(T)) = \bar{r}(f^{\leftarrow}(T))$.
- (2) If f is bijective, then $f^{\leftarrow}(\bar{s}(T)) = \bar{s}(f^{\leftarrow}(T))$.
- (3) If f is surjective, then $f^{\leftarrow}(ar(T)) = ar(f^{\leftarrow}(T))$.
- (4) $f^{\leftarrow}(s(T)) = s(f^{\leftarrow}(T))$.

Proof. (1) By Definition 4.1 and Theorem 5.10(1), $f^{\leftarrow}(\bar{r}(T))$ is a reflexive soft set relation on (F, A) . By Theorem 4.3(1), $\bar{r}(T) = T \cup I \supset T$. According to Theorem 5.7(1), $f^{\leftarrow}(\bar{r}(T)) \supset f^{\leftarrow}(T)$.

Next, we will show that if V is a reflexive soft set relation on (F, A) and $V \supset f^{\leftarrow}(T)$, then $f^{\leftarrow}(\bar{r}(T)) \subset V$. Note that f is bijective. By Theorems 5.8(2) and 5.6(1), $T = f^{\rightarrow}(f^{\leftarrow}(T)) \subset f^{\rightarrow}(V)$ since $f^{\leftarrow}(T) \subset V$, i.e. $T \subset f^{\rightarrow}(V)$. On the other hand, by Theorem 5.9(1), $f^{\rightarrow}(V)$ is a reflexive soft set relation on (G, B) . Then $\bar{r}(T) \subset f^{\rightarrow}(V)$. According to Theorems 5.8(1) and 5.7(1), $f^{\leftarrow}(\bar{r}(T)) \subset f^{\leftarrow}(f^{\rightarrow}(V)) = V$, i.e. $f^{\leftarrow}(\bar{r}(T)) \subset V$.

So $f^{\leftarrow}(\bar{r}(T)) = \bar{r}(f^{\leftarrow}(T))$.

(2) It is similar to (1).

(3) By Definition 3.9(1) and Theorem 5.10(2), $f^{\leftarrow}(ar(T))$ is an anti-reflexive soft set relation on (F, A) . By Theorem 3.10(1), $ar(T) = T \cap T^c \subset T$. According to Theorem 5.7(1), $f^{\leftarrow}(ar(T)) \subset f^{\leftarrow}(T)$.

Next, we will show that if V is an anti-reflexive soft set relation on (F, A) and $V \subset f^{\leftarrow}(T)$, then $f^{\leftarrow}(ar(T)) \supset V$. By Theorems 5.8(2) and 5.6(1), $T \supset f^{\rightarrow}(f^{\leftarrow}(T)) \supset f^{\rightarrow}(V)$ since $f^{\leftarrow}(T) \supset V$, i.e. $T \supset f^{\rightarrow}(V)$. On the other hand, by Theorem 5.9(3), $f^{\rightarrow}(V)$ is an anti-reflexive soft set relation on (G, B) . Then $ar(T) \supset f^{\rightarrow}(V)$. According to Theorems 5.7(1) and 5.8(1), $f^{\leftarrow}(ar(T)) \supset f^{\leftarrow}(f^{\rightarrow}(V)) \supset V$, i.e. $f^{\leftarrow}(ar(T)) \supset V$.

So $f^{\leftarrow}(ar(T)) = ar(f^{\leftarrow}(T))$.

(4) It is similar to (3). \square

The following example is presented to illustrate the basic ideas developed in Theorem 5.13.

Example 5.14. Let $U = \{p_1, p_2, p_3, p_4, p_5\}$, $A = \{a_1, a_2\}$, and $B = \{b_1, b_2\}$. Consider the soft sets (F, A) and (G, B) defined by $F(a_1) = \{p_1, p_2, p_5\}$, $F(a_2) = \{p_1, p_3, p_4\}$, and $G(b_1) = \{p_1, p_3, p_4\}$, $G(b_2) = \{p_2, p_5\}$. A soft set function f from (F, A) to (G, B) is given by $f = \{F(a_1) \times G(b_1), F(a_2) \times G(b_2)\}$. Obviously, f is bijective. Let T be a soft set relation on (G, B) given by $T = \{G(b_1) \times G(b_1)\}$. Then

$$\bar{r}(T) = T \cup I = \{G(b_1) \times G(b_1), G(b_2) \times G(b_2)\},$$

$$f^{\leftarrow}(\bar{r}(T)) = \{F(a_1) \times F(a_1), F(a_2) \times F(a_2)\}.$$

Thus $\bar{r}(f^{\leftarrow}(T)) = \{F(a_1) \times F(a_1), F(a_2) \times F(a_2)\}$.

Similarly, we can obtain $\bar{s}(f^{\leftarrow}(T))$, $s(f^{\leftarrow}(T))$ and $ar(f^{\leftarrow}(T))$ by using $f^{\leftarrow}(\bar{s}(T))$, $f^{\leftarrow}(s(T))$ and $f^{\leftarrow}(ar(T))$, respectively.

6. Conclusion

In the present paper the theoretical point of view of soft sets is discussed again. We construct kernels and closures of soft set relations, and soft set relation mappings by the use of soft set relations and functions. Some related properties are discussed. We also give some examples to explain these theories. Based on these results, we can further probe the applications of soft sets. Moreover, the concepts proposed in this paper can be extended in fuzzy soft sets and thus one can get more affirmative solution in decision making problems in real life situations.

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References

- [1] D. Molodtsov, Soft set theory—first results, *Comput. Math. Appl.* 37 (1999) 19–31.
- [2] F. Feng, Y.B. Jun, X. Liu, L. Li, An adjustable approach to fuzzy soft set based decision making, *J. Comput. Appl. Math.* 234 (1) (2010) 10–20.
- [3] N. Çağman, S. Enginoğlu, Soft matrix theory and its decision making, *Comput. Math. Appl.* 59 (10) (2010) 3308–3314.
- [4] N. Çağman, S. Enginoğlu, Soft set theory and *uni-int* decision making, *European J. Oper. Res.* 207 (2) (2010) 848–855.
- [5] Z. Kong, L. Gao, L. Wang, Comment on “A fuzzy soft set theoretic approach to decision making problems”, *J. Comput. Appl. Math.* 223 (2) (2009) 540–542.
- [6] P.K. Maji, A.R. Roy, R. Biswas, An application of soft sets in a decision making problem, *Comput. Math. Appl.* 44 (8–9) (2002) 1077–1083.
- [7] A.R. Roy, P.K. Maji, A fuzzy soft set theoretic approach to decision making problems, *J. Comput. Appl. Math.* 203 (2) (2007) 412–418.
- [8] Y. Zou, Z. Xiao, Data analysis approaches of soft sets under incomplete information, *Knowl.-Based Syst.* 21 (8) (2008) 941–945.
- [9] Z. Xiao, K. Gong, Y. Zou, A combined forecasting approach based on fuzzy soft sets, *J. Comput. Appl. Math.* 228 (1) (2009) 326–333.
- [10] S.J. Kalayathankal, G.S. Singh, A fuzzy soft flood alarm model, *Math. Comput. Simulation* 80 (5) (2010) 887–893.
- [11] Bozena Kostek, Soft set approach to the subjective assessment of sound quality, in: *IEEE Conferences*, vol. 1, 1998, pp. 669–674.
- [12] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, *Comput. Math. Appl.* 45 (2003) 555–562.
- [13] P.K. Maji, R. Biswas, A.R. Roy, Fuzzy soft sets, *J. Fuzzy Math.* 9 (3) (2001) 589–602.
- [14] P. Majumdar, S.K. Samanta, Generalised fuzzy soft sets, *Comput. Math. Appl.* 59 (4) (2010) 1425–1432.
- [15] P.K. Maji, More on intuitionistic fuzzy soft sets, in: H. Sakai, M.K. Chakraborty, A.E. Hassanien, D. Slezak, W. Zhu (Eds.), *Proceedings of the 12th International Conference on Rough Sets, Fuzzy Sets, Data Mining and Granular Computing, RSFDGrC 2009*, in: *Lecture Notes in Computer Science*, vol. 5908, Springer, 2009, pp. 231–240.
- [16] P.K. Maji, R. Biswas, A.R. Roy, Intuitionistic fuzzy soft sets, *J. Fuzzy Math.* 9 (3) (2001) 677–692.
- [17] P.K. Maji, A.R. Roy, R. Biswas, On intuitionistic fuzzy soft sets, *J. Fuzzy Math.* 12 (3) (2004) 669–683.
- [18] W. Xu, J. Ma, S. Wang, G. Hao, Vague soft sets and their properties, *Comput. Math. Appl.* 59 (2) (2010) 787–794.
- [19] X.B. Yang, T.Y. Lin, J.Y. Yang, Y. Li, D. Yu, Combination of interval-valued fuzzy set and soft set, *Comput. Math. Appl.* 58 (3) (2009) 521–527.
- [20] Y. Jiang, Y. Tang, Q. Chen, H. Liu, J. Tang, Interval-valued intuitionistic fuzzy soft sets and their properties, *Comput. Math. Appl.* 60 (3) (2010) 906–918.
- [21] H. Aktaş, N. Çağman, Soft sets and soft groups, *Inform. Sci.* 177 (13) (2007) 2726–2735.
- [22] A. Aygunoglu, H. Aygun, Introduction to fuzzy soft groups, *Comput. Math. Appl.* 58 (6) (2009) 1279–1286.
- [23] F. Feng, Y.B. Jun, X.Z. Zhao, Soft semirings, *Comput. Math. Appl.* 56 (2008) 2621–2628.
- [24] U. Acar, F. Koyuncu, B. Tanay, Soft sets and soft rings, *Comput. Math. Appl.* 59 (11) (2010) 3458–3463.
- [25] Y.B. Jun, Soft BCK/BCI-algebras, *Comput. Math. Appl.* 56 (5) (2008) 1408–1413.
- [26] Y.B. Jun, C.H. Park, Applications of soft sets in ideal theory of BCK/BCI-algebras, *Inform. Sci.* 178 (11) (2008) 2466–2475.
- [27] Y.B. Jun, K.J. Lee, J. Zhan, Soft *p*-ideals of soft BCI-algebras, *Comput. Math. Appl.* 58 (10) (2009) 2060–2068.
- [28] Y.B. Jun, K.J. Lee, C.H. Park, Soft set theory applied to ideals in *d*-algebras, *Comput. Math. Appl.* 57 (3) (2009) 367–378.
- [29] Y.B. Jun, K.J. Lee, A. Khan, Soft ordered semigroups, *Math. Logic Q.* 56 (1) (2010) 42–50.
- [30] J. Zhan, Y.B. Jun, Soft BL-algebras based on fuzzy sets, *Comput. Math. Appl.* 59 (6) (2010) 2037–2046.
- [31] Z. Xiao, K. Gong, S. Xia, Y. Zou, Exclusive disjunctive soft sets, *Comput. Math. Appl.* 59 (6) (2010) 2128–2137.
- [32] K. Gong, Z. Xiao, X. Zhang, The bijective soft set with its operations, *Comput. Math. Appl.* 60 (8) (2010) 2270–2278.
- [33] Y. Jiang, Y. Tang, Q. Chen, J. Wang, S. Tang, Extending soft sets with description logics, *Comput. Math. Appl.* 59 (6) (2010) 2087–2096.
- [34] M.I. Ali, F. Feng, X. Liu, W.K. Min, M. Shabir, On some new operations in soft set theory, *Comput. Math. Appl.* 57 (9) (2009) 1547–1553.
- [35] K. Qin, Z. Hong, On soft equality, *J. Comput. Appl. Math.* 234 (5) (2010) 1347–1355.
- [36] A. Kharal, B. Ahmad, Mappings on fuzzy soft classes, *Adv. Fuzzy Syst.* (2009) 6. Article ID 407890.
- [37] K.V. Babitha, J.J. Sunil, Soft set relations and functions, *Comput. Math. Appl.* 60 (7) (2010) 1840–1849.
- [38] K. Hrbacek, T. Jech, *Introduction to Set Theory*, Marcel Dekker Inc., 1984.
- [39] S. Patrick, *Axiomatic Set Theory*, D. Van Nostrand Company Inc., 1960.
- [40] D. Pei, D. Miao, From soft sets to information systems, in: *Granular Computing, IEEE International Conference*, vol. 2, 2005, pp. 617–621.
- [41] J. Kelley, *General Topology*, Van Nostrand Company, 1955.